MATH 1650 SUMMARY OF THE CONIC SECTIONS

GEOMETRIC DEFINITION: 'Conic sections' come from slicing the cone at different angles:

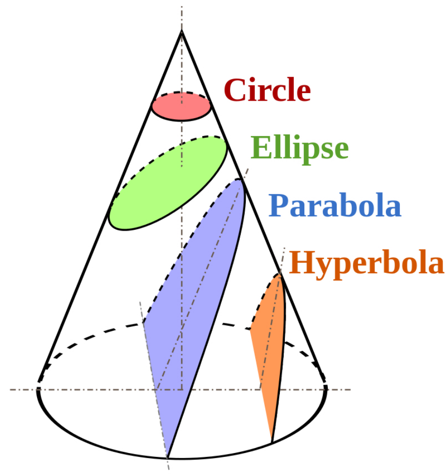


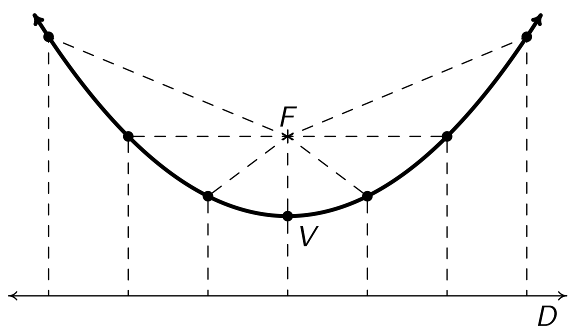
Image taken from Wikipedia.

They can also be described as a **locus** (set) of points in the plane which satisfy certain distance conditions.

ALGEBRAIC DEFINITION: The conic sections arise from graphing quadratic equations in two variables:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

PARABOLAS: Given a fixed line called the **directrix**, D, and a fixed point not on the line called the **focus**, F, a **parabola** is the set of all points whose distance to the directrix is the same as their distance to the focus.



Equations of parabolas have only **one** squared term, and there are two standard forms:

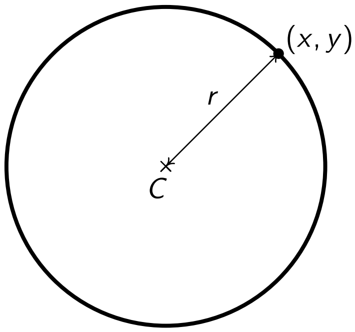
$(x-h)^2 = 4p(y-k)$ $\qquad$ $(y-k)^2 = 4p(x-h)$

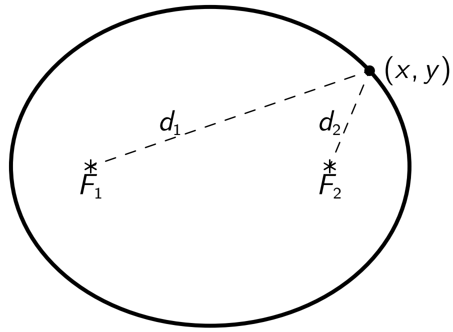
$p > 0$ $\qquad$ $p < 0$ $\qquad$ $p > 0$ $\qquad$ $p < 0$

In each case:

- the vertex is (h, k).
- p is the *directed* distance from the vertex to the focus (and from the vertex to the directrix.)
- $|4p|$ is the width of the parabola at the focus (the length of the latus rectum)

CIRCLES AND ELLIPSES: Given a fixed point called the **center**, C, and a fixed distance called the **radius**, r, a **circle** is the set of all points r units away from the center. Given two fixed points called **foci**, F_1 and F_2, and a fixed distance d, an **ellipse** is the set of all points the **sum** of whose distances to the foci is d.

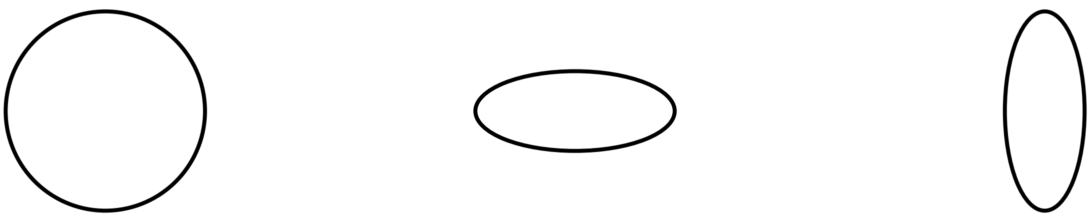




$d_1 + d_2 = d$ for all (x, y) on the ellipse

Equations of circles and ellipses contain **two** squared terms with the **same** signs. Equal coefficients point to a circle; unequal coefficients indicate an ellipse. The standard form is:

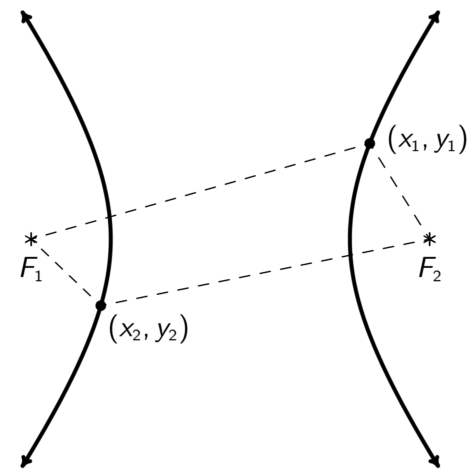
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



$a = b$ $\qquad$ $a > b$ $\qquad$ $b > a$

- The center in each case is (h, k).
- 'a' tells us how far to move in x-direction from the center (i.e., the horizontal stretch.)
- 'b' tells us how far to move in the y-direction from the center. (i.e., the vertical stretch.)
- If $a = b$, the curve is a circle, and $a = b = r$ is the radius of the circle.
- If $a > b$, the curve is an ellipse with a horizontal major axis.
  - The vertices are 'a' units to the left and right of the center.
  - The foci are $c = \sqrt{a^2 - b^2}$ units to the left and right of the center.
- If $b > a$, the curve is an ellipse with a vertical major axis.
  - The vertices are 'b' units above and below the center.
  - The foci are $c = \sqrt{b^2 - a^2}$ units above and below the center.
- The eccentricity, $e = \dfrac{\text{distance from center to focus}}{\text{distance from center to vertex}} = \dfrac{c}{a \text{ or } b\text{, whichever is bigger}}$
  - e is a measure of 'roundness' and for ellipses, $0 < e < 1$.
  - If $e \approx 0$, the ellipse is more circular; if $e \approx 1$, the ellipse is less circular (more 'eccentric.')

HYPERBOLAS: Given two fixed points called **foci**, F_1 and F_2, and a fixed distance d, a **hyperbola** is the set of all points the **difference** of whose distances to the foci is d.



In the figure above:

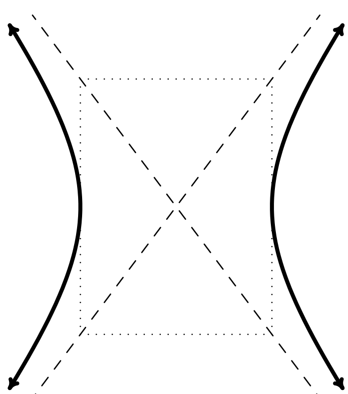
$$\text{the distance from } F_1 \text{ to } (x_1, y_1) - \text{the distance from } F_2 \text{ to } (x_1, y_1) \;=\; d$$

and

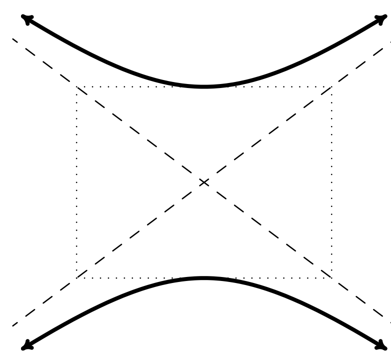
$$\text{the distance from } F_2 \text{ to } (x_2, y_2) - \text{the distance from } F_1 \text{ to } (x_2, y_2) \;=\; d$$

Equations of hyperbolas contain **two** squared terms with **different** signs. There are two standard forms:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$



- The center in each case is (h, k).
- To sketch a 'guide rectangle:' from the center, move 'a' units left and right and 'b' units up and down.
- The slant asymptotes contain contain the center, (h, k) and have slopes $\pm b/a$.
- If the standard form contains $x^2 - y^2$, the curve is a hyperbola with a horizontal transverse axis.
  - The vertices are 'a' units to the left and right of the center.
  - The foci are $c = \sqrt{a^2 + b^2}$ units to the left and right of the center.
- If the standard form contains $y^2 - x^2$, the curve is a hyperbola with a vertical transverse axis.
  - The vertices are 'b' units above and below the center.
  - The foci are $c = \sqrt{b^2 + a^2}$ units above and below the center.